# 1.6: Absolute Value Functions

## How to Solve an Absolute Value Equation

**Isolate:** Isolate the absolute the absolute value term to have the form $|A| = B$. If $B < 0$, No solution exists.

**Two Equations:** If $B \geq 0$, use $|A| = B$ to write two equations $A = B$ and $A = -B$.

**Solve:** for $x$.

**Extraneous Solutions:** Plug in the $x$ previously found into the original equation to eliminate the extraneous solutions.

This method works for equations of the type $|A| = |B|$ as well.

Now, you can complete Problems 1-4.

## How to Solve Absolute Value Inequalities

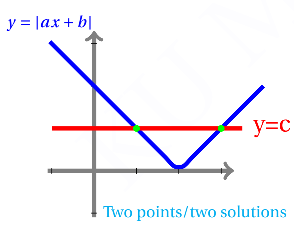
- Isolate the absolute value term.
- Rewrite the inequality into two inequalities.
  If $\boxed{|A| < B}$, then $-B < A < B$. If $\boxed{|A| > B}$, then $A > B$ or $A < -B$.
- When solving $|ax - b| < c$,

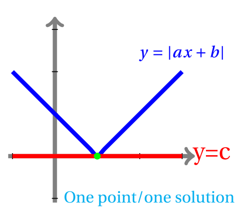

- When solving $|ax - b| > c$.

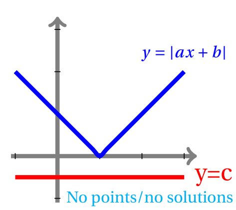

Now, you can complete Problems 5 and 6.

## Illustration of Absolute Value Equations

- Solving an equation of the form of $|ax + b| = c$ can be viewed as finding the intersection of graph of function $y = |ax + b|$ and $y = c$. So if we assume $B$ is constant, we will have three cases:




## The Absolute Value Function is Piecewise-defined

- $|x| = \begin{cases} x & \text{when } x \geq 0 \\ -x & \text{when } x < 0 \end{cases}$ (Check this fact by taking a sample value in each rule.)
- $|x - a| = \begin{cases} x - a & \text{when } x - a \geq 0 \\ -(x - a) & \text{when } x - a < 0 \end{cases} \Longrightarrow |x - a| = \begin{cases} x - a & x \geq a \\ -x + a & x < a \end{cases}$

Now, you can complete Problems 7 and 8.

1. Solve $7|11x-4|+1=22$ for $x$.

2. Solve $-11|7x-4|+46=13$ for $x$.

3. Solve $|11x+7|=12x+4$ for $x$.

4. Solve equation $|x|=|2x+6|$ for $x$.

5. **One of the important applications of absolute value functions is in finding the bounds of measurement error.** A factory is producing bags of snacks that are labeled as $150\,gr$. The producer is required to keep the actual weight within 2% of the labeled weight. Let $w$ be the actual weight of a bag in grams. Write the range of the actual weight as an absolute value inequality.

6. Solve each of these inequalities.

   (a) $|11x - 2| < 6$

   (b) $|11x - 2| \geq 3$

7. Consider the function $g(x) = -|3x - 11| + 4$.

(a) Identify the parent function and describe the transformation on $g$ (shifts, stretches, etc).

(b) Use this description to sketch a graph of $g$.

8. Consider $f(x) = |x - 4| + 5$.

(A) For what values of $x$ is $y = x - 4$ positive? For what values of $x$ is $y = x - 4$ negative?

(B) Rewrite $f$ as a piecewise-defined function. Explain what information you used from Part (A).

(C) Graph $y = f(x)$.